Access to Science, Engineering and Agriculture: Mathematics 1
MATH00030
Chapter 1 Solutions

1. (a) $\dfrac{3}{7} - \dfrac{3}{8} = \dfrac{(3)(8) + (-3)(7)}{(7)(8)} = \dfrac{3}{56}$.

   (b) $\dfrac{3}{5} + \dfrac{1}{9} = \dfrac{(3)(9) + (1)(5)}{(5)(9)} = \dfrac{32}{45}$.

   (c) $3 - \dfrac{10}{11} = \dfrac{(3)(11) - 10}{11} = \dfrac{23}{11}$.

   (d) $\dfrac{1}{2} + \dfrac{2}{3} + \dfrac{4}{5} = \dfrac{(1)(3)(5) + (2)(2)(5) + (4)(2)(3)}{(2)(3)(5)} = \dfrac{59}{30}$.

2. (a) $\dfrac{2}{3} \times \dfrac{4}{7} = \dfrac{2 \times 4}{3 \times 7} = \dfrac{8}{21}$.

   (b) $\dfrac{8}{7} \times \left(-\dfrac{1}{3}\right) = \dfrac{8 \times (-1)}{7 \times 3} = \dfrac{-8}{21} = -\dfrac{8}{21}$.

   (c) $-\dfrac{5}{4} \times \left(-\dfrac{4}{5}\right) = \dfrac{(-5) \times (-4)}{4 \times 5} = \dfrac{20}{20} = 1$.

   (d) $4 \times \left(-\dfrac{1}{4}\right) = \dfrac{4 \times (-1)}{1 \times 4} = \dfrac{-4}{4} = -1$.

   (e) $\dfrac{2}{3} \div \dfrac{1}{3} = \dfrac{2}{3} \times \dfrac{3}{1} = \dfrac{2 \times 3}{3 \times 1} = \dfrac{6}{3} = 2$.

   (f) $-\dfrac{6}{7} \div \left(-\dfrac{7}{6}\right) = -\dfrac{6}{7} \times \left(-\dfrac{6}{7}\right) = \dfrac{-6}{7} \times \dfrac{-6}{7} = \dfrac{-6 \times (-6)}{7 \times 7} = \dfrac{36}{49}$.

   (g) $\dfrac{10}{3} \div \left(-\dfrac{2}{7}\right) = \dfrac{10}{3} \times \left(-\dfrac{7}{2}\right) = \dfrac{10}{3} \times \dfrac{-7}{2} = \dfrac{10 \times (-7)}{3 \times 2} = \dfrac{-70}{6} = -\dfrac{35}{3}$.

   (h) $0 \div 1 = \dfrac{0}{1} = 0$.

   (i) $1 \div 0$ does not equal anything.

3. (a) $6 \div 7 \times 8 + 9 = 6 \times \dfrac{1}{7} \times 8 + 9 = \dfrac{6}{7} \times 8 + 9 = \dfrac{48}{7} + 9 = \dfrac{48 + 63}{7} = \dfrac{111}{7}$.

   (b) $6 \div 7 \times (8 + 9) = 6 \div 7 \times 17 = 6 \times \dfrac{1}{7} \times 17 = \dfrac{6}{7} \times 17 = \dfrac{102}{7}$.

   (c) $6 \div (7 \times 8 + 9) = 6 \div (56 + 9) = 6 \div 65 = \dfrac{6}{65}$.

   (d) $6 \div (7 \times 8) + 9 = 6 \div 56 + 9 = \dfrac{6}{56} + 9 = \dfrac{3}{28} + 9 = \dfrac{3 + 252}{28} = \dfrac{255}{28}$.

4. (a) $3^3 = 3 \times 3 \times 3 = 27$.

   (b) $(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) = -32$.

   (c) $\left(\frac{1}{2}\right)^4 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$.

   (d) $\sqrt{16} = 4$.
   Note -4 is **NOT** correct.

   (e) $\sqrt[3]{64} = 4$.

   (f) $\sqrt[15]{1} = 1$.

   (g) $(64)^{\frac{2}{3}} = \left(\sqrt[3]{64}\right)^2 = 4^2 = 16$.

   (h) $(16)^{-\frac{3}{2}} = \frac{1}{(16)^{\frac{3}{2}}} = \frac{1}{\left(\sqrt{16}\right)^3} = \frac{1}{4^3} = \frac{1}{64}$.

   (i) $\left(\frac{4}{25}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{4}{25}}\right)^3 = \left(\frac{2}{5}\right)^3 = \frac{8}{125}$.

   (j) $\left(\frac{27}{8}\right)^{-\frac{5}{3}} = \frac{1}{\left(\frac{27}{8}\right)^{\frac{5}{3}}} = \frac{1}{\left(\sqrt[3]{\frac{27}{8}}\right)^5} = \frac{1}{\left(\frac{3}{2}\right)^5} = \frac{1}{243/32} = \frac{32}{243}$.

5. (a) $x^9 \times x^6 = x^{9+6} = x^{15}$.

   (b) $x^{10} \times x^{-13} = x^{10+(-13)} = x^{-3}$.

   (c) $x^{\frac{3}{4}} \times x^{\frac{1}{3}} = x^{\frac{3}{4}+\frac{1}{3}} = x^{\frac{9+4}{12}} = x^{\frac{13}{12}}$.

   (d) $x^{\frac{1}{2}} \times x^{-\frac{3}{4}} = x^{\frac{1}{2}+\left(-\frac{3}{4}\right)} = x^{\frac{2-3}{4}} = x^{-\frac{1}{4}}$.

   (e) $(x^3)^4 = x^{3\times 4} = x^{12}$.

   (f) $(x^{-3})^2 = x^{-3\times 2} = x^{-6}$.

   (g) $\left(x^{\frac{1}{2}}\right)^{-\frac{1}{3}} = x^{\frac{1}{2}\times\left(-\frac{1}{3}\right)} = x^{-\frac{1}{6}}$.

   (h) $x^6 \div x^4 = x^{6-4} = x^2$.

   (i) $x^{\frac{1}{2}} \div x^{-\frac{2}{3}} = x^{\frac{1}{2}-\left(-\frac{2}{3}\right)} = x^{\frac{3+4}{6}} = x^{\frac{7}{6}}$.

   (j) $\left(x^{-\frac{1}{3}} \times x^{-\frac{1}{2}}\right)^{\frac{3}{2}} = \left(x^{-\frac{1}{3}+\left(-\frac{1}{2}\right)}\right)^{\frac{3}{2}} = \left(x^{\frac{-2-3}{6}}\right)^{\frac{3}{2}} = \left(x^{-\frac{5}{6}}\right)^{\frac{3}{2}} = x^{-\frac{5}{6}\times\frac{3}{2}} = x^{-\frac{15}{12}}$
   $= x^{-\frac{5}{4}}$.

6. (a) $5 \times 4^2 = 5 \times 16 = 80$.

   (b) $(5 \times 4)^2 = 20^2 = 400$.

   (c) $5 \div 2^3 + 4 = 5 \div 8 + 4 = 5 \times \frac{1}{8} + 4 = \frac{5}{8} + 4 = \frac{5+32}{8} = \frac{37}{8}$.

   (d) $5 \div (2^3 + 4) = 5 \div (8 + 4) = 5 \div 12 = \frac{5}{12}$.

   (e) $(5 \div 2)^3 + 4 = \left(\frac{5}{2}\right)^3 + 4 = \frac{5^3}{2^3} + 4 = \frac{125}{8} + 4 = \frac{125+32}{8} = \frac{157}{8}$.

(f) $3\times4\div5+2^2 = 3\times4\div5+4 = 3\times4\times\frac{1}{5}+4 = 12\times\frac{1}{5}+4 = \frac{12}{5}+4 = \frac{12+20}{5} = \frac{32}{5}$.

(g) $3\times4\div(5+2)^2 = 3\times4\div7^2 = 3\times4\div49 = 3\times4\times\frac{1}{49} = 12\times\frac{1}{49} = \frac{12}{49}$.

(h) $3\times(4\div5+2)^2 = 3\times\left(\frac{4}{5}+2\right)^2 = 3\times\left(\frac{4+10}{5}\right)^2 = 3\times\left(\frac{14}{5}\right)^2 = 3\times\frac{196}{25}$
$= \frac{588}{25}$.

(i) $(3\times4\div5+2)^2 = \left(3\times4\times\frac{1}{5}+2\right)^2 = \left(12\times\frac{1}{5}+2\right)^2 = \left(\frac{12}{5}+2\right)^2$
$= \left(\frac{12+10}{5}\right)^2 = \left(\frac{22}{5}\right)^2 = \frac{484}{25}$.

(j) $3\times(4\div5+2^2) = 3\times(4\div5+4) = 3\times\left(\frac{4}{5}+4\right) = 3\times\left(\frac{4+20}{5}\right) = 3\times\frac{24}{5} = \frac{72}{5}$.

7. (a) $\left(x^2\sqrt[3]{y}\right)^3 = \left(x^2\right)^3\left(\sqrt[3]{y}\right)^3 = \left(x^2\right)^3\left(y^{\frac{1}{3}}\right)^3 = x^{2(3)}y^{\frac{1}{3}(3)} = x^6y^1 = x^6y$.

(b) $\left(x^{-3}y^{\frac{1}{2}}\right)^{\frac{2}{3}} = \left(x^{-3}\right)^{\frac{2}{3}}\left(y^{\frac{1}{2}}\right)^{\frac{2}{3}} = x^{-3\left(\frac{2}{3}\right)}y^{\frac{1}{2}\left(\frac{2}{3}\right)} = x^{-\frac{6}{3}}y^{\frac{2}{6}} = x^{-2}y^{\frac{1}{3}}$.

(c) $\left(x^{-4}y^{-\frac{2}{3}}\right)^{-2} = \left(x^{-4}\right)^{-2}\left(y^{-\frac{2}{3}}\right)^{-2} = x^{-4(-2)}y^{-\frac{2}{3}(-2)} = x^8y^{\frac{4}{3}}$.

(d) $\left(xy^{-\frac{1}{3}}z^{\frac{1}{2}}\right)^6 = \left(\left(x^1y^{-\frac{1}{3}}\right)z^{\frac{1}{2}}\right)^6 = \left(x^1y^{-\frac{1}{3}}\right)^6\left(z^{\frac{1}{2}}\right)^6 = \left(x^1\right)^6\left(y^{-\frac{1}{3}}\right)^6\left(z^{\frac{1}{2}}\right)^6$
$= x^{1(6)}y^{-\frac{1}{3}(6)}z^{\frac{1}{2}(6)} = x^6y^{-\frac{6}{3}}z^3 = x^6y^{-2}z^3$.

8. (a) Since $4^2 = 16$, it follows that $\log_4 16 = 2$.

(b) Since $5^3 = 125$, it follows that $\log_5 125 = 3$.

(c) Since $36^{\frac{1}{2}} = 6$, it follows that $\log_{36} 6 = \frac{1}{2}$.

(d) Since $20^{-1} = \frac{1}{20}$, it follows that $\log_{20}\frac{1}{20} = -1$.

(e) Since $8^{-2} = \frac{1}{64}$, it follows that $\log_8\frac{1}{64} = -2$.

(f) Since $27^{-\frac{1}{3}} = \frac{1}{3}$, it follows that $\log_{27}\frac{1}{3} = -\frac{1}{3}$.

9. (a) $\log_a\left(x^4y^{\frac{1}{2}}\right) = \log_a\left(x^4\right) + \log_a\left(y^{\frac{1}{2}}\right) = 4\log_a(x) + \frac{1}{2}\log_a(y)$.

(b) $\log_a\left(\left(\frac{x^2}{y^3}\right)^{-2}\right) = -2\log_a\left(\frac{x^2}{y^3}\right) = -2\left(\log_a\left(x^2\right) - \log_a\left(y^3\right)\right)$
$= -2\left(2\log_a(x) - 3\log_a(y)\right) = -4\log_a(x) + 6\log_a(y)$.
Note that alternatively we could use $\left(\frac{x^2}{y^3}\right)^{-2} = \frac{x^{-4}}{y^{-6}} = \frac{y^6}{x^4}$ and then simplify using the rules of logarithms.

(c) $\log_a\left(x^{\log_a\left(y^2\right)}\right) = \log_a\left(y^2\right)\left(\log_a x\right) = 2(\log_a y)(\log_a x)$.
Or alternatively $\log_a\left(x^{\log_a\left(y^2\right)}\right) = \log_a\left(x^{2\log_a y}\right) = 2(\log_a y)(\log_a x)$.

10. (a) $15.450 = 15.5$ to one decimal place.

    (b) $9.95 = 10.0$ to one decimal place.

    (c) $0.004 = 0.00$ to two decimal places.

    (d) 10.000 to three decimal places.

    (e) $-1.56 = -1.6$ to one decimal place.

    (f) $-10.655 = -10.65$ to two decimal places.

11. (a) $7595462381 = 7600000000$ to three significant figures.

    (b) $0.000125 = 0.00013$ to two significant figures.

    (c) $29.95 = 30$ to two significant figures.

    (d) $30 = 30.00$ to four significant figures.

    (e) $-1.45 = -1.4$ to two significant figures.

    (f) $-0.01216 = -0.0122$ to three significant figures.

12. (a) $14674.45 = 1.467445 \times 10^4$ in scientific notation.

    (b) $0.00436 = 4.36 \times 10^{-3}$ in scientific notation.

    (c) $43543.4445 = 4.35 \times 10^4$ in scientific notation to three significant figures.

    (d) $0.00345 = 3.450 \times 10^{-3}$ in scientific notation to four significant figures.

13. (a) $(2x^3 - 2x^2 + 3x - 4) + (-x^3 + 3x + 4) = (2x^3 - x^3) - 2x^2 + (3x + 3x) + (-4 + 4)$
    $= x^3 - 2x^2 + 6x.$

    (b) $(-3x^3 - 5x + 7) - (-4x^3 + 3x^2 - 3x + 9)$
    $= (-3x^3 - (-4x^3)) - 3x^2 + (-5x - (-3x)) + (7 - 9)$
    $= x^3 - 3x^2 - 2x - 2.$

    (c) $(3x^7 + 3x^3 - 2x^{-1} + 4x^{-4}) + (7x^3 + 7 - x^{-1} - 3x^{-4})$
    $= 3x^7 + (3x^3 + 7x^3) + 7 + (-2x^{-1} - x^{-1}) + (4x^{-4} - 3x^{-4})$
    $= 3x^7 + 10x^3 + 7 - 3x^{-1} + x^{-4}.$

14. (a)

$$\begin{aligned} 3x^3(x^2 - 3x + 3) &= (3x^3)(x^2) + (3x^3)(-3x) + (3x^3)(3) \\ &= 3x^{3+2} + (3)(-3)x^{3+1} + 9x^3 \\ &= 3x^5 - 9x^4 + 9x^3. \end{aligned}$$

    (b)

$$\begin{aligned} (x^2 + 3x)(-3x^2 + 5) &= (x^2)(-3x^2 + 5) + (3x)(-3x^2 + 5) \\ &= (x^2)(-3x^2) + (x^2)(5) + (3x)(-3x^2) + (3x)(5) \\ &= -3x^{2+2} + 5x^2 + (3)(-3)x^{1+2} + 15x \\ &= -3x^4 + 5x^2 - 9x^3 + 15x \\ &= -3x^4 - 9x^3 + 5x^2 + 15x. \end{aligned}$$

(c)

$$\begin{aligned}(4x-2)(x^2+4x+1) =&(4x)(x^2+4x+1)+(-2)(x^2+4x+1)\\ =&(4x)(x^2)+(4x)(4x)+(4x)(1)\\ &+(-2)(x^2)+(-2)(4x)+(-2)(1)\\ =&4x^{1+2}+(4)(4)x^{1+1}+4x-2x^2-8x-2\\ =&4x^3+16x^2+4x-2x^2-8x-2\\ =&4x^3+14x^2-4x-2.\end{aligned}$$

(d)

$$\begin{aligned}(4x^2-x+1)(-x^2-x-1) =&(4x^2)(-x^2-x-1)+(-x)(-x^2-x-1)\\ &+(1)(-x^2-x-1)\\ =&(4x^2)(-x^2)+(4x^2)(-x)+(4x^2)(-1)\\ &+(-x)(-x^2)+(-x)(-x)+(-x)(-1)\\ &+(1)(-x^2)+(1)(-x)+(1)(-1)\\ =&(4)(-1)x^{2+2}+(4)(-1)x^{2+1}-4x^2\\ &+(-1)(-1)x^{1+2}+(-1)(-1)x^{1+1}+x-x^2-x-1\\ =&-4x^4-4x^3-4x^2+x^3+x^2+x-x^2-x-1\\ =&-4x^4-3x^3-4x^2-1.\end{aligned}$$

(e)

$$\begin{aligned}(-x^{-1}-2x^{-2})(x^{-1}+3x^{-2}) =&(-x^{-1})(x^{-1}+3x^{-2})+(-2x^{-2})(x^{-1}+3x^{-2})\\ =&(-x^{-1})(x^{-1})+(-x^{-1})(3x^{-2})\\ &+(-2x^{-2})(x^{-1})+(-2x^{-2})(3x^{-2})\\ =&(-1)x^{-1-1}+(-1)(3)x^{-1-2}\\ &+(-2)x^{-2-1}+(-2)(3)x^{-2-2}\\ =&-x^{-2}-3x^{-3}-2x^{-3}-6x^{-4}\\ =&-x^{-2}-5x^{-3}-6x^{-4}.\end{aligned}$$

15. (a)

$$\begin{array}{r} 10535 \\ 6\overline{)\,63211} \\ \underline{60000} \\ 3211 \\ \underline{3000} \\ 211 \\ \underline{180} \\ 31 \\ \underline{30} \\ 1 \end{array}$$

So $\dfrac{63211}{6} = 10535 + \dfrac{1}{6}$.

That is the quotient is 10535 and the remainder is 1.

(b)
$$\begin{array}{r} 64912 \\ 5\overline{)324563} \\ \underline{300000} \\ 24563 \\ \underline{20000} \\ 4563 \\ \underline{4500} \\ 63 \\ \underline{50} \\ 13 \\ \underline{10} \\ 3 \end{array}$$

So $\dfrac{324563}{5} = 64912 + \dfrac{3}{5}$.

That is the quotient is 64912 and the remainder is 3.

(c)
$$\begin{array}{r} 24941 \\ 23\overline{)573653} \\ \underline{460000} \\ 113653 \\ \underline{92000} \\ 21653 \\ \underline{20700} \\ 953 \\ \underline{920} \\ 33 \\ \underline{23} \\ 10 \end{array}$$

So $\dfrac{573653}{23} = 24941 + \dfrac{10}{23}$.

That is the quotient is 24941 and the remainder is 10.

(d)
$$\begin{array}{r} 89013 \\ 521\overline{)46375835} \\ \underline{41680000} \\ 4695835 \\ \underline{4689000} \\ 6835 \\ \underline{5210} \\ 1625 \\ \underline{1563} \\ 62 \end{array}$$

So $\dfrac{46375835}{521} = 89013 + \dfrac{62}{521}$.

That is the quotient is 89013 and the remainder is 62.

16. (a)
$$\begin{array}{r|l} & x-2 \\ \hline x+1 & x^2-x+1 \\ & -x^2-x \\ \hline & -2x+1 \\ & 2x+2 \\ \hline & 3 \end{array}$$

This tells us that $\dfrac{x^2-x+1}{x+1} = x-2+\dfrac{3}{x+1}$.

So the quotient is $x-2$ and the remainder is 3.

(b)
$$\begin{array}{r|l} & x^2+2 \\ \hline x-1 & x^3-x^2+2x+2 \\ & -x^3+x^2 \\ \hline & 2x+2 \\ & -2x+2 \\ \hline & 4 \end{array}$$

This tells us that $\dfrac{x^3-x^2+2x+1}{x-1} = x^2+2+\dfrac{4}{x-1}$.

So the quotient is x^2+2 and the remainder is 4.

(c)
$$\begin{array}{r|l} & x^2-2x+1 \\ \hline 3x+1 & 3x^3-5x^2+x-2 \\ & -3x^3-x^2 \\ \hline & -6x^2+x \\ & 6x^2+2x \\ \hline & 3x-2 \\ & -3x-1 \\ \hline & -3 \end{array}$$

This tells us that $\dfrac{3x^3-5x^2+x-2}{3x+1} = x^2-2x+1+\dfrac{-3}{3x+1}$.

So the quotient is x^2-2x+1 and the remainder is -3.

(d)
$$\begin{array}{r|l} & 2x^2-2x-5 \\ \hline x^2+x+1 & 2x^4-5x^2+x-2 \\ & -2x^4-2x^3-2x^2 \\ \hline & -2x^3-7x^2+x \\ & 2x^3+2x^2+2x \\ \hline & -5x^2+3x-2 \\ & 5x^2+5x+5 \\ \hline & 8x+3 \end{array}$$

This tells us that $\dfrac{2x^4-5x^2+x-2}{x^2+x+1} = 2x^2-2x-5+\dfrac{8x+3}{x^2+x+1}$.

So the quotient is $2x^2-2x-5$ and the remainder is $8x+3$.

17. (a) $\displaystyle\sum_{i=1}^{5} i = 1+2+3+4+5 = 15.$

(b) $\displaystyle\sum_{i=0}^{4} i^3 = 0^3+1^3+2^3+3^3+4^3 = 0+1+8+27+64 = 100.$

(c) $\displaystyle\sum_{i=-2}^{2} 2i^2 = 2(-2)^2+2(-1)^2+2(0)^2+2(1)^2+2(2)^2 = 8+2+0+2+8 = 20.$

18. (a) $\displaystyle\sum_{i=-2}^{1} x^i = x^{-2}+x^{-1}+x^0+x^1 = x^{-2}+x^{-1}+1+x.$

(b) $\displaystyle\sum_{i=0}^{4} x^{2i} = x^0+x^2+x^4+x^6+x^8 = 1+x^2+x^4+x^6+x^8.$

(c) $\displaystyle\sum_{i=-2}^{2} ix^3 = -2x^3-x^3+(0)x^3+x^3+2x^3 = 0.$

19. (a) $\dbinom{11}{2} = \dfrac{11\times 10}{2} = 55.$

(b) $\dbinom{20}{3} = \dfrac{20\times 19\times 18}{3\times 2} = 1140.$

(c) $\dbinom{88}{86} = \dbinom{88}{2} = \dfrac{88\times 87}{2} = 3828.$

(d) $\dbinom{100}{100} = 1.$

20. (a) $(x+y^2)^2 = x^2+\dbinom{2}{1}xy^2+(y^2)^2 = x^2+2xy^2+y^4.$

(b) $(2x+3y)^2 = (2x)^2+\dbinom{2}{1}(2x)(3y)+(3y)^2 = 4x^2+12xy+9y^2.$

(c) $(2+3y)^3 = 2^3+\dbinom{3}{1}2^2(3y)+\dbinom{3}{2}(2)(3y)^2+(3y)^3 = 8+36y+54y^2+27y^3.$

(d)

$$\begin{aligned}(3x+y^3)^3 &= (3x)^3+\binom{3}{1}(3x)^2(y^3)+\binom{3}{2}(3x)(y^3)^2+(y^3)^3\\ &= 27x^3+27x^2y^3+9xy^6+y^9.\end{aligned}$$